Technical Section

STATISTICAL CHARACTERIZATION OF THE EEG: THE USE OF THE POWER SPECTRUM AS A MEASURE OF ERGODICITY

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Summary In order to interpret statistical summaries of treatment-related changes in EEG power spectra, the internal consistency of the set of data from each condition must first be known. A technique for the assessment of such across-record homogeneity (ergodicity) is derived from the defining characteristics of the power spectrum. A simplified graphic approximation is also detailed. Computer simulations and practical validation confirm it as a useful means of evaluating the ergodicity of sets of EEG data.

Keywords: power spectrum – ergodicity

Frequency analysis procedures are useful for the evaluation of state changes induced in the EEG by a variety of manipulations. For example, changes in the frequency composition of the EEG have been studied after the administration of drugs (Benignus et al. 1974; Vanderwolf 1975; Gilden and Kozakiewicz 1976), the electrical stimulation of discrete brain loci (Anchel and Lindsley 1972; Paiva et al. 1976; Tenke 1984), and spontaneous changes in behavior (Vanderwolf 1975; Leung et al. 1982). In order to test changes in power spectra with standard gaussian statistical tools, normalizing transformations are required (Bendat and Piersol 1971; Gasser et al. 1982). These procedures are generally acceptable only as long as the transformations are adequate, the data are unimodal and the variances are comparable across conditions. Non-parametric procedures, which are less affected by departures from a gaussian distribution, have also been applied to spectral data (Leung et al. 1982). The latter procedures, however, may be incapable of resolving differences between multimodal distributions.

The EEG is clearly capable of supporting distinct, persistent modes of activity (e.g., alpha vs. low voltage fast activity). If these bimodal trends are present in a set of data, standard statistical techniques will lead to unwarranted conclusions unless the homogeneity of the data is also assessed. Time domain segmenting procedures have already been developed to divide EEG data into stationary records based on various homogeneity requirements (Praetorius et al. 1977; Michael and Houchin 1979). In a comparable fashion, between-record homogeneity may be viewed as an indication of the ergodicity of a signal. The aim of this paper is to develop a means of assessing this type of variability in the EEG.

Frequency domain artifactual procedures have been described (Gevins et al. 1977) which exclude from a set of EEG data those records which are extremely deviant in their spectral structure. In a comparable fashion, the technique developed here examines the scatter of the smoothed estimates of the power spectrum (power spectral density, PSD) as a means of assessing the ergodicity of a set of EEG data. The capacity to subdivide a multimodal set of EEG data into homogeneous subsets is also illustrated.

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Derivation

Theoretical data structure and test ratio

The PSD may be considered as a frequency-specific representation of the statistical variance of the signal. It formally denotes the density of the power of a signal as a function of frequency. The former interpretation is true because the average power (mean of squared amplitude) is identical to the variance whenever the signal offset (mean) is zero.

For the digital computation of the PSD, an estimate \( \hat{G}_X(f, T) \) is computed at each frequency point (f) for a signal (X) of duration T. Each estimate is produced from the squared magnitude of the value at each frequency point yielded by Fourier transformation (FT) of the complete data record. Because the FT is linear, if the underlying signal amplitude (voltage) distribution is gaussian, the error distribution at each frequency point must follow the chi-squared distribution

\[
\frac{\hat{G}_X(f, T)}{G_X(f)} = \chi^2_f
\]

where \( G_X(f) \) represents the population mean (Bendat and Piersol 1971). The number of degrees of freedom (\( df \)) is 2 because amplitude and phase may vary independently.

In order to assess the likelihood that the PSD of a single EEG record is part of a gaussian process common to a complete set of data, it is necessary to first reduce the variability of each estimate by increasing its \( df \). Standard smoothing procedures combine estimates across frequency and/or across data records. The distribution of the sum of \( \chi^2 \) distributions is itself \( \chi^2 \) with its \( df \) equal to the sum of those of the component distributions (e.g., Hays and Winkler 1971). The error distribution of such a pooled estimate \( \hat{G}_1 \) is distributed as \( \chi^2_{df}/df \), where \( df \) is the sum of the \( df \) of all contributing estimates (Bendat and Piersol 1971). The ratio of the error distributions of two mean estimates (\( \hat{G}_{x1} \) and \( \hat{G}_{x2} \)) of the same population mean \( G_X \) is therefore given by

\[
\frac{\hat{G}_{x1}(f, T)/G_X(f)}{\hat{G}_{x2}(f, T)/G_X(f)} = \frac{\chi^2_{df1}/df1}{\chi^2_{df2}/df2} = F_{df1, df2}
\]

The population means cancel to leave a simple ratio of means. The implementation of eqn 2 as a test of interrecord stability simply requires the separation of individual record means from the remaining data.

The generation of the test ratio may be briefly summarized as follows. A mean estimate of the PSD \( \hat{G}_X(f, T) \) is produced from each raw PSD estimate by averaging (smoothing) across frequency. For each value, the \( df \) is twice the number of consecutive frequency points contributing to the average. The denominator of eqn 2 is the corresponding reference group value, which is the grand mean across records of all remaining smoothed power spectra in the set. The corresponding \( df \) is simply twice the total number of all raw PSD estimates contributing to it.

Evaluation of bias error

Frequency smoothing procedures inevitably introduce a bias error into the smoothed PSD estimate that is related to the second derivative of the underlying mean unsmoothed spectrum being estimated (Bendat and Piersol 1971). This distortion...
tion occurs whenever the value of the underlying population mean $G_0(f)$ changes drastically across a frequency smoothing band since a single smoothed mean $\tilde{G}_0(f)$ may not be adequate to approximate the value of $G_0(f)$ at the midpoint of this band. The distortion introduced into the test ratio by a log linear (i.e., exponential) variation over frequency was simulated on a microcomputer. Simulated PSD estimates were smoothed over 20 points to mimic 1 Hz smoothing of records 20 sec long. Sets of the corresponding test ratio values were derived from 20 such independent estimates.

The test ratio was found to be virtually unaffected by log slopes of up to 0.5 Hz. The distributions of biased and unbiased test ratios consistently diverged only when log slopes exceeded 1. The effect of a large bias (1.5 log units/Hz) on the extremes of the expected test ratio is contrasted to that of a small (0.3 log unit) discontinuity across records in Fig. 1. Fig. 2A illustrates a sample EEG characterized by a spectral peak with a log slope of 0.87/Hz as a physiological reference. The simulations show that the test ratio is resistant to the bias error typical of the EEG, but sensitive to the bimodal interrecord inhomogeneities expected when ergodicity lapses.

Application

Critical values of test ratio

Each data record contributes a series of independent frequency estimates to the smoothed PSD. When evaluating the stability of an entire spectrum, each data record will therefore have an inflated capacity to contribute an extreme estimate. In Fig. 1, the maximum and minimum of each set of 20 ratios from unbiased simulated data are ranked and plotted at the midpoint of the corresponding probability interval.

To test only one of a 20 record set of 20-point frequency-smoothed values, the medians (0.5 probability) of the simulated unbiased distributions may be used as critical ratios. These extremes are (approximately) equivalent to the tabled pair of critical $F$ ratios ($df = 40$ and 760; $P = 0.025$ each tail). To simultaneously test a range of 20 such smoothed estimates, the simulated 0.05 and 0.95 probability extremes may be used. In general, for a given number of consecutive smoothed frequency estimates $n_e$, the appropriate critical ratio value may be obtained from Fig. 1 at probabilities of $1/(n_e + 1)$ and $1 - 1/(n_e + 1)$.

Upon rejection of a data record, it should be noted that the $df$ associated with the group mean will be decreased. In practice, the change in the critical values for the log of the test ratio is small as long as the $df$ remain large. The sample rate, the frequency band over which the data are smoothed, and the total number of samples for each EEG record all affect the $df$ for each estimate in a predictable fashion. It is therefore advisable to assess the symmetry and stability of the

![Figure 2](image)

Fig. 2. Representative bipolar hippocampal EEGs. A: sample 10.24 sec data record illustrates the rhythmicity of an EEG with a power spectrum peak characterized by a log slope of 0.87/Hz. Cosine taper function is apparent at beginning and end of recording period. Animal was restrained (Tenke et al., 1983) and subjected to brain-stem stimulation (Tenke 1984). 200 μV calibration. B: representative 20.48 sec data record from those used to validate the test procedure. 230 μV calibration.
logs of the paired critical $F$ distribution values whenever these parameters are changed.

**Graphic implementation**

If the smoothed power spectra have been plotted on a logarithmic scale, the test ratio of eqn 2 may be applied by directly measuring the difference between the log of the smoothed PSD estimate for one record and the log of the mean of the remaining spectra. As a screening procedure, this test may be approximated by measuring the range (in log units) of the set of smoothed spectra without actually computing the means for the reference groups. If the variability of the data markedly exceeds that expected by chance, those records contributing isolated extreme estimates should be immediately rejected. After this, individual records contributing extremes at a number of different frequencies may be rejected.

When blatantly artifactual records have been eliminated, the variability of the data may be reassessed. If all estimates now occupy the predicted range, the data set may be accepted as homogeneous. If the range is frequently exceeded, the data should briefly be assessed for multimodal trends. Such trends may be used to divide the data into homogeneous subgroups. Otherwise, the few remaining outliers should be eliminated in a stepwise fashion, from the most to the least extreme. Failure to achieve homogeneity graphically may indicate the use of other sorting methods, such as a frequency domain adaptation of the one described by Ruchkin (1971).

**Validation**

**Subject and recording procedure**

The bipolar EEG of the dorsal hippocampus of a restrained rat was used to assess the applicability of eqn 1 to real data records of moderate length (20.48 sec). Simultaneous recordings from the lateral hypothalamus served as an independent means of assessing the homogeneity of the data set. The EEGs were recorded on FM tape by LC converters after amplification by a differential amplifier (0.06–60 Hz bandpass with a 60 Hz notch). The use of an additional off-line 30 Hz low-pass filter and a 200 sample/sec digitization rate served as an antialiasing procedure. The digitized data records were cosine-tapered over their first and last tenths after the removal of any residual DC bias (Bendat and Piersol 1971). PSD estimates were obtained after Fourier transformation and pooled across non-overlapping 0.98 Hz bands. A total of 20 consecutive records were combined to produce 400 total estimates/band (20 estimates/band for each record).

**Applicability of gaussian model of the EEG**

Fig. 2B shows a representative record from the obtained set of EEG data. The scatter of the 20 individual smoothed power spectra obtained is illustrated in Fig. 3. As is evident from Figs. 2B and 3, a degree of rhythmicity was present in this EEG. The estimate centered at 6.32 Hz reflects the activity at the single spectral peak which is largely confined to this band.

The distribution of all raw PSD estimates contributing to the smoothed spectral peak (pooled within and across records) is displayed in Fig. 4 with a comparable plot of $\log \left( \frac{\chi^2}{\text{median } \chi^2} \right)$. As is readily apparent from Fig. 4, the estimates corresponding to the spectral peak (solid circles) closely followed the theoretical distribution. While
some portions of the spectrum do not conform as well to a gaussian model (the 1.44 Hz and 15.11 Hz bands plotted in Fig. 4), the stability of the 6.32 Hz peak suggests the feasibility of a subdivision of the data into one or more ergodic subsets.

Sample application to EEG data

For ergodic data with a minimal bias error, Fig. 1 indicates that the range of the power ratio for 20 independent frequencies should not exceed 0.7 log units (2.5 ±1). Graphically applying this criterion to Fig. 3, few estimates below 8 Hz are in violation of homogeneity. A single data record contributes the smallest estimates across the 3.4–5.3 Hz and 8.3–11.2 Hz ranges. Another record contributes the largest values at 0.5 and 5.3 Hz. After the exclusion of these two records, the rising slope of the theta peak (4–8 Hz) is consistent with that predicted from Fig. 1 (expected 0.5 log unit span for 4 independent points). Slower activity (<4 Hz), however, remains somewhat variable. Although a third data record contributes the remaining highest values across the entire low frequency range (not shown), activity below 1.4 Hz is still somewhat more variable than expected.

In contrast to the relative homogeneity of activity below 8 Hz, the high amplitude PSD estimates at 15.1 Hz reflect the presence of a secondary peak in a subset of data that is distinctly bimodal at 17.1 Hz (Fig. 3). Records containing this secondary peak contributed the largest estimates to the falling slope of the theta peak as well. In these records, simultaneous recordings from the hypothalamus exhibited a corresponding spectral peak. In portions of the spectrum, these records were readily separable from the remaining data by graphic procedures (dashed lines in Fig. 3). Use of a runs test verified this as a trend in the temporal order of the data.

The power spectra of Fig. 3 may be readily resolved into two distinct subsets by graphic means alone. These subgroups are consistent with the gaussian model of eqn 1 for all but the slowest frequency bands. The sensitivity of the test procedure to interrecord inhomogeneities (Fig. 1) is therefore of practical value.

Discussion

The effect of an experimental treatment (e.g., drug administration) on the EEG PSD cannot be assessed without making simplifying assumptions about the statistical structure of the individual data sets. While it is widely recognized that the assumptions underlying gaussian statistics may be violated by inhomogeneities in the data, the so-called distribution free techniques are also susceptible to the distorting influence of a multimodal distribution. The simplicity with which the homogeneity of a data set may be assessed by the test ratio described here makes it a valuable adjunct in the evaluation of treatment-related changes in the EEG power spectra.

The validation data suggest that most of the EEG spectrum may be reduced to component gaussian processes. For very low frequency activity, however, this description may fail. Gasser et al. (1982) have also noted an inconsistency in the structure of the PSD in the delta band. The current data suggest that the division of data sets into ergodic or quasieergodic (a slight discrepancy confined to a small portion of the spectrum) data subsets will provide the most fundamental summary of a set of EEG data.

Routine adoption of the technique described
here would require little effort, particularly if the
power spectra have already been log transformed
for other purposes. In return, a knowledge of the
homogeneity of the data would improve the inter-
pretability of statistical summaries which might
otherwise be inexplicable. It would also facilitate
the investigation of simultaneous changes in the
quality, quantity and incidence of elementary EEG
patterns.

Résumé

Caractérisation statistique de l’EEG: utilisation du
spectre de puissance comme mesure d’ergodicité

Afin d’interpréter des résumés statistiques con-
cernant des modifications du spectre de puissance
de l’EEG liées à un traitement, la stabilité des
données correspondant à chaque échantillon doit
d’abord être établie. Une technique pour estimer
une telle homogénéité entre enregistrements
(ergodicité) est établie à partir des caractéristiques
de définition du spectre de puissance. Une
approximation graphique simplifiée est également
présentée. Des simulations sur ordinateurs et une
validation pratique ont confirmé son utilité comme
moyen d’évaluation de l’ergodicité d’ensemble de
données EEG.

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